GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC TASK 2

2024

Instructions:

- **Working time – 90 minutes.**
- **Write using black/blue pen.**
- **Calculators approved by NESA may be used.**
- **A reference sheet is provided.**
- **For section 1, use the multiple-choice answer sheet provided. Attempt questions 1 - 5.**
- **For Questions in Section II, show relevant mathematical reasoning and/or calculations. Use the answer booklet provided. Attempt questions 6 – 11.**
- **Marks may be deducted for careless or badly arranged work.**

Section 1 (5 marks)

Questions 1 – 5

Use the multiple-choice answer sheet for questions 1 to 5

1. For what value(s) of a are the vectors $\underset{\sim}{p} = \begin{pmatrix} 3a-7 \\ 2 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} a \\ 1 \end{pmatrix}$ perpendicular?

(A) $a = -\frac{1}{3}$ and $a = 2$

(B) $a = \frac{1}{3}$ and $a = -2$

(C) $a = \frac{1}{3}$ and $a = 2$

(D) $a = -\frac{1}{3}$ and $a = -2$

2. Joey and Abi start at the origin and walk along the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$ and then turn and walk along the vector $4\underset{\sim}{i} - 2\underset{\sim}{j}$. How far is Joey and Abi from the origin?

(A) 5

(B) $\sqrt{11}$

(C) $\sqrt{21}$

(D) $\sqrt{37}$

3. A vector $\underset{\sim}{u}$ is defined as $\underset{\sim}{u} = 6\underset{\sim}{i} - 8\underset{\sim}{j}$. Which of the following represent the unit vector in the direction of $\underset{\sim}{u}$?

(A) $\hat{\underset{\sim}{u}} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$

(B) $\hat{\underset{\sim}{u}} = \underset{\sim}{i} - \underset{\sim}{j}$

(C) $\hat{\underset{\sim}{u}} = \frac{3}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

(D) $\hat{\underset{\sim}{u}} = \frac{3}{4}\underset{\sim}{i} - \frac{4}{3}\underset{\sim}{j}$

4. Which one of the following vectors is parallel to the vector $\overrightarrow{OP} = -2\underset{\sim}{i} + \underset{\sim}{j}$

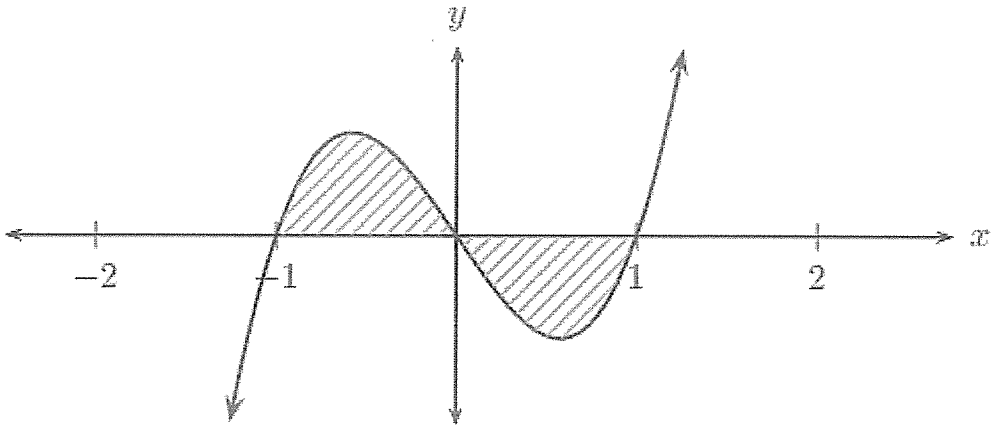
(A) $\overrightarrow{OA} = -12\underset{\sim}{i} + 8\underset{\sim}{j}$

(B) $\overrightarrow{OB} = 12\underset{\sim}{i} - 6\underset{\sim}{j}$

(C) $\overrightarrow{OC} = -\underset{\sim}{i} + 2\underset{\sim}{j}$

(D) $\overrightarrow{OD} = 2\underset{\sim}{i} + \underset{\sim}{j}$

5. The diagram shows the area bounded by the graph of $f(x) = x^3 - x$ and the x axis.



Given that $f(x)$ is an odd function, which of the following gives the correct expression for the shaded area?

(A) $2\int_{-1}^{0} (x^3 - x)\, dx$

(B) $2\int_{0}^{1} (x^3 - x)\, dx$

(C) $\int_{-1}^{1} (x^3 - x)\, dx$

(D) $2\int_{-1}^{1} (x^3 - x)\, dx$

Section II (82 marks)

- Answer questions in the answer booklet provided.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks)

(a) Consider the function $f(x) = 7 + 4x^3 - 3x^4$

(i) Find the stationary points and determine their nature. 4

(ii) Find any points of inflexion. 3

(iii) Sketch the curve showing $y-$intercept, stationary points and points of inflexion. 3

(b) Given $\underset{\sim}{a} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} -5 \\ -12 \end{pmatrix}$ and $\underset{\sim}{c} = m\underset{\sim}{i} + n\underset{\sim}{j}$. Calculate:

(i) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ 1

(ii) the exact angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ correct to the nearest degree. 2

(iii) If $\underset{\sim}{a} \cdot \underset{\sim}{c} = \underset{\sim}{b} \cdot \underset{\sim}{c}$, find m in terms of n. 2

Please turn over

Question 7 (14 marks)

(a) The position vectors of the points P and Q are given by $\underset{\sim}{p} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}, \underset{\sim}{q} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$.

(i) Find $\overrightarrow{PQ}$. 2

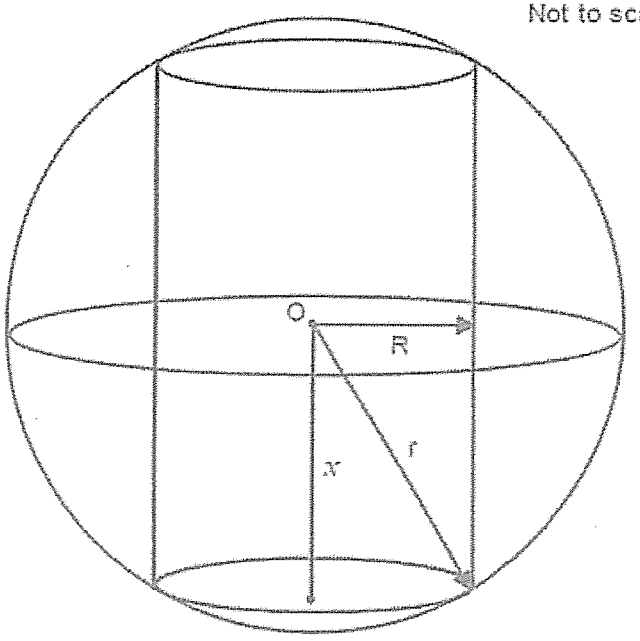
(ii) Calculate the magnitude and direction of $\overrightarrow{PQ}$ (anticlockwise from the positive x-axis) 3

(b) (i) Find $\dfrac{d}{dx}\left(\dfrac{7}{2x+x^3}\right)$ 2

(ii) Hence find $\displaystyle\int \frac{2+3x^2}{(2x+x^3)^2}\,dx$ 2

(c) A cylinder is to be made to fit inside a sphere of radius r *cm*, as shown. Let R be the radius of the cylinder and x be the distance from the centre of the sphere to the base of the cylinder as shown in the diagram below.

Not to scale



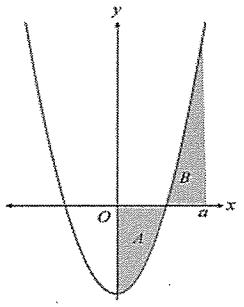
(i) Find an expression for the radius of the cylinder (R) in terms of r and x. 1

(ii) Show that the volume V, of the cylinder is given by $V = 2\pi x(r^2 - x^2)$ 1

(iii) Find in terms of r, the maximum volume of the cylinder. Leave your answer in exact form. 3

Question 8 (13 marks)

(a) The graph of $y = x^2 - 4$ is shown below. The area of the region A is equal to the area of the region B. What is the value of a? 3



Not to scale

(b) A curve has a stationary point at (-1,1). If the gradient is given by $6x^2 + kx - 12$,

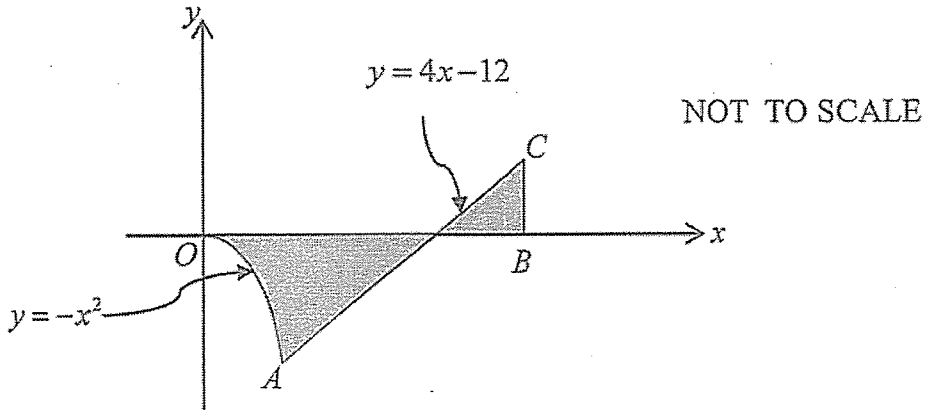
Find:

(i) the value of k. 1

(ii) the equation of the curve. 2

(c) Use Trapezoidal rule with five function values to find an approximation to the value of $\int_0^2 3^x \, dx$. Give your answer correct to 3 decimal places. 3

(d) The shaded region is bounded by the line $x = 0, x = 4$, the curve $y = -x^2$, the line $y = 4x - 12$ and the x axis as in the diagram. A has co-ordinates (2, -4).

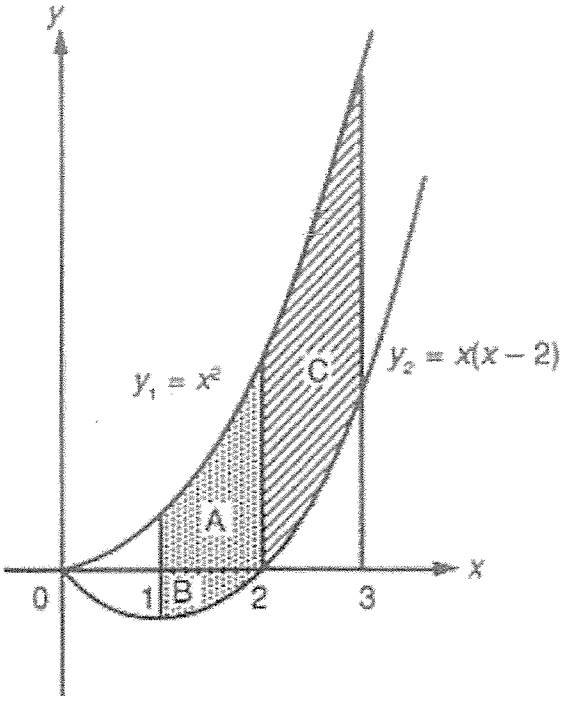


What is the area of the shaded region? 4

Question 9 (13 marks)

(a) The diagram shows the curves $y_1 = x^2$, $y_2 = x^2 - 2x$ and three shaded sections A, B and C . Calculate the areas of:

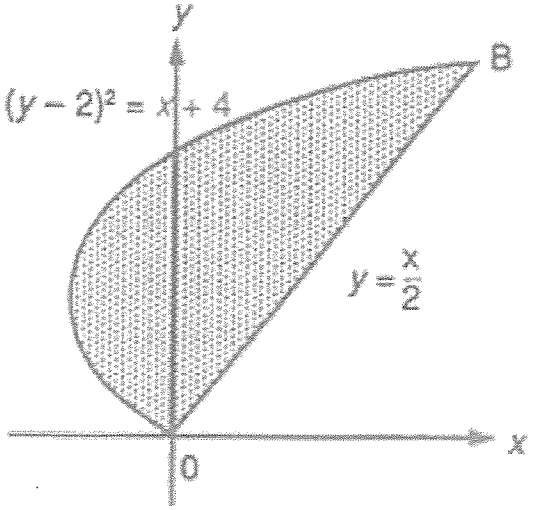
(i) A **(ii)** B **(iii)** C (2+3+3)



(b) The region bounded by the parabola $(y-2)^2 = x+4$ and the line $y = \frac{x}{2}$ is shown in the diagram below. Find:

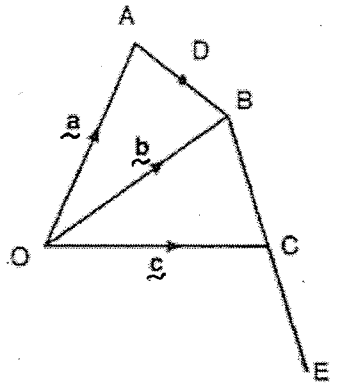
(i) the coordinates of B . 2

(ii) Calculate the shaded area. 3



Question 10 (12 marks)

(a) $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are the vectors $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively. D is the midpoint of AB and E lies on BC where $BE = 2BC$. Write the following in terms of $\underset{\sim}{a}, \underset{\sim}{b}$ and $\underset{\sim}{c}$.



(i) $\overrightarrow{AD}$

(ii) $\overrightarrow{OD}$

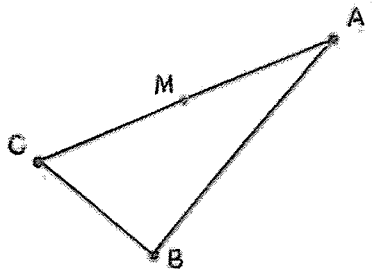
(iii) $\overrightarrow{BC}$

(iv) $\overrightarrow{BE}$

(v) $\overrightarrow{OE}$

(vi) $\overrightarrow{DE}$ 6

(b) ΔABC is a right-angled triangle with M being the midpoint of the hypotenuse AC, as shown below. Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



(i) Copy the diagram into your answer booklet showing all the given information 1

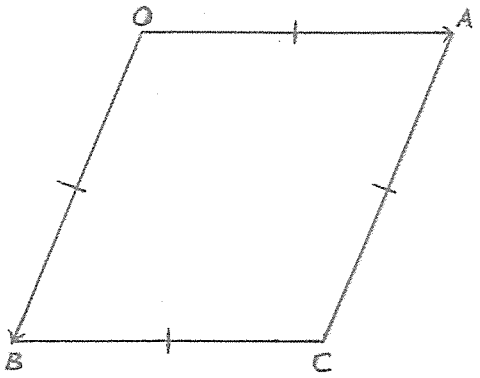
(ii) Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. 2

(iii) Hence prove that M is equidistant from the three vertices of ΔABC. 3

Question 11 (10 marks)

(a) Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 4\underset{\sim}{j}$. Find the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. 3

(b) $OACB$ is a rhombus with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.



(i) Copy the diagram onto your answer booklet and draw the vector representing $\underset{\sim}{a} + \underset{\sim}{b}$ 1

(ii) Write an expression for $\cos \angle AOC$ using dot product of vectors. 2

(iii) Hence prove that the diagonal OC bisects $\angle AOB$. 4

End of Examination

Please remember to write 'End of Solutions' on your answer paper.

Year 12 Extension 1 Task 2 2024 Solutions

Multiple choice (5 marks)

1. $\underset{\sim}{p} \cdot \underset{\sim}{q} = 0$

$a(3a-1) + 2 = 0$

$3a^2 - 7a + 2 = 0$ (C)

$(a-2)(3a-1) = 0$

$a = 2, \quad a = \frac{1}{3}$

2. $2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{i} - 2\underset{\sim}{j}$

$= 6\underset{\sim}{i} + \underset{\sim}{j}$ (D)

magnitude $= \sqrt{6^2 + 1^2}$

$= \sqrt{37}$

3. $\hat{\underset{\sim}{u}} = \frac{6\underset{\sim}{i} - 8\underset{\sim}{j}}{\sqrt{6^2 + 8^2}}$

$= \frac{6\underset{\sim}{i} - 8\underset{\sim}{j}}{10}$

$= \frac{6}{10}\underset{\sim}{i} - \frac{8}{10}\underset{\sim}{j}$ (C)

$= \frac{3}{5}\underset{\sim}{i} - \frac{4}{5}\underset{\sim}{j}$

4. $-6(-2\underset{\sim}{i} + \underset{\sim}{j})$

$= 12\underset{\sim}{i} - 6\underset{\sim}{j}$ (B)

5. (A)

A few students got this one wrong.

Marker's comments:

Question 6 (15 marks)

(a)(i) $f(x) = 7 + 4x^3 - 3x^4$

$f'(x) = 12x^2 - 12x^3$

$f''(x) = 24x - 36x^2$

$f'(x) = 0 \Rightarrow 12x^2(1-x) = 0$

$x = 0$ or $x = 1$

when $x = 0$, $y = 7$

when $x = 1$, $y = 8$

stationary points are $(0,7)$ and $(1,8)$

$f''(0) = 0$

$f''(1) = (24 \times 1) - (36 \times 1)$

$= -12 < 0$ (4)

$\therefore (1,8)$ is a maximum turning point

(ii) Possible points of inflexion are given by $f''(x) = 0$

$24x - 36x^2 = 0$

$12x(2 - 3x) = 0$

$x = 0$ or $x = \frac{2}{3} = 0.7$

Marker's comments:

Generally well done.

Comments for (a)

PTO →

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
f''	3	0	-21

U ∩

$\therefore (0,7)$ is a horizontal point of inflexion

x	0.6	$\frac{2}{3}=0.7$	0.8
f''	1.44	0	-3.84

(3)

U ∩

when $x=\frac{2}{3}$, $y=7.6$

$$f\left(\frac{2}{3}\right) = 7 + 4\left(\frac{2}{3}\right)^3 - \left(3\times\left(\frac{2}{3}\right)^4\right)$$

$$= 7.6$$

$\therefore (0.7, 7.6)$ is a point of inflexion.

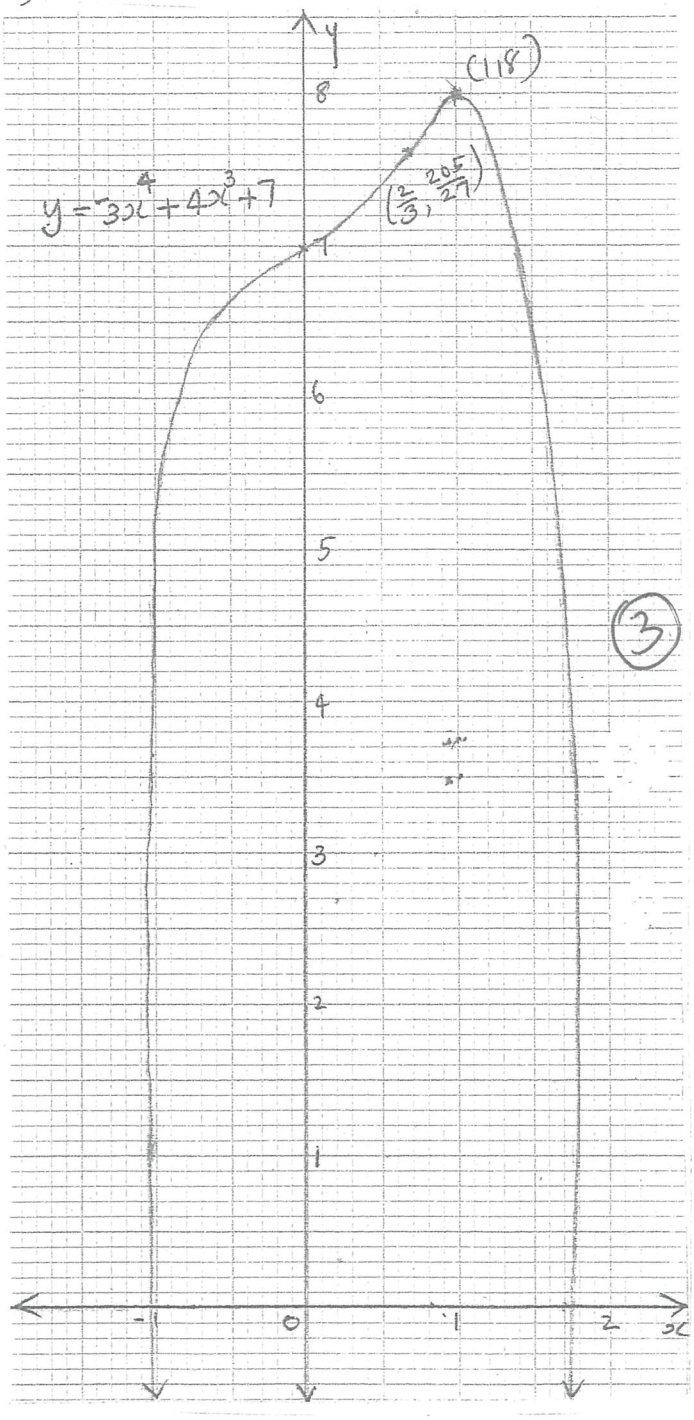
or $\left(\frac{2}{3}, 7\frac{16}{27}\right)$ or $\left(\frac{2}{3}, \frac{205}{27}\right)$

Marker's comments:

(ii) Tests of double derivative between points poorly done. Often for $(0,7)$ students would test ✗ $x=-1$ ✓ & 1 ↑ which is no good as the other point of inflexion was at $x=\frac{2}{3}$.

They needed to test BETWEEN $x=0$ & $x=\frac{2}{3}$.

(iii)



3

Marker's comments:

Generally well done.

Carry over marks were awarded for students who made mistakes in (i) and (ii) but graphed their results correctly.

(b) (i)

$\underset{\sim}{a} \cdot \underset{\sim}{b} = (2 \times -5) + (-5 \times -12)$

$= -10 + 60 = 50$ (1)

(ii) $\cos\theta = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|\,|\underset{\sim}{b}|}$

$= \dfrac{50}{\sqrt{2^2+5^2} \cdot \sqrt{5^2+12^2}}$

$= \dfrac{50}{13\sqrt{29}}$

$\theta = 44^\circ$ (2)

(iii) $\underset{\sim}{a} \cdot \underset{\sim}{c} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} m \\ n \end{pmatrix} = 2m - 5n$

$\underset{\sim}{b} \cdot \underset{\sim}{c} = \begin{pmatrix} -5 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} m \\ n \end{pmatrix} = -5m - 12n$

$\underset{\sim}{a} \cdot \underset{\sim}{c} = \underset{\sim}{b} \cdot \underset{\sim}{c}$

$-5m - 12n = 2m - 5n$ (2)

$-5m - 2m = 12n - 5n$

$-7m = 7n$

$m = -n$ or $m + n = 0$

Marker's comments:

Well done.

Well done.

Generally very well done.

A small minority of students did

$\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{b} \cdot \underset{\sim}{c}$

$50 = -5m - 12n$ etc.

which made an "easier" [& wrong] question.

Also some students wrote

$2m\underset{\sim}{i} - 5n\underset{\sim}{j} = -5m\underset{\sim}{i} - 12n\underset{\sim}{j}$

This was not penalised BUT DOT PRODUCT RESULTS IN A NUMBER - NOT A VECTOR!

It's just $2m - 5n = -5m - 12n$.

Question 7 (14 marks)

(a) (i) $\overrightarrow{PQ} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ (2)

$= \begin{pmatrix} -4-2 \\ -1+4 \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

(ii) $|\overrightarrow{PQ}| = \sqrt{6^2+3^2}$

$= \sqrt{36+9}$

$= \sqrt{45}$

$= 3\sqrt{5}$ (3)

$\tan\theta = \frac{3}{6} = \frac{1}{2}$

direction $= 180 + \tan^{-1}(\frac{1}{2})$

$= 180 - 27^\circ$

$= 153^\circ$

(b)(i) $\frac{d}{dx}\left(\frac{7}{2x+x^3}\right)$

$= \frac{(2x+x^3) \times 0 - 7(2+3x^2)}{(2x+x^3)^2}$

$= \frac{-7(2+3x^2)}{(2x+x^3)^2}$ (2)

From (i) we have

$\int \frac{-7(2+3x^2)}{(2x+x^3)^2}\,dx = \frac{7}{2x+x^3} + C$

$-7\int \frac{(2+3x^2)}{(2x+x^3)^2}\,dx = \frac{7}{2x+x^3} + C$

Markers comments:

7a i, Some students made calculation errors other than that most students got this part correct

7a ii, if students made errors in a i, then they got the magnitude & direction wrong.

Overall most mistakes were ~~found~~ made in the finding the direction of the vector.

7b i, some students forgot the negative sign, which shows that they might not be using the correct order while applying the Quotient rule.

$f'(x) = \frac{u'v - uv'}{v^2}$

b ii, well done! All students have shown good understanding of reverse Chain rule.

6

$$\int \frac{2+3x^2}{(2x+x^3)^2}\,dx$$

$$= \frac{-1}{7} \times \frac{7}{2x+x^3} + C \quad (2)$$

$$= \frac{-1}{2x+x^3} + C$$

(c)(i) By Pythagoras' theorem

$$r^2 = R^2 + x^2$$

$$R^2 = r^2 - x^2 \quad (1)$$

$$R = \sqrt{r^2 - x^2} \quad (\because R > 0)$$

(ii) $V = \pi R^2 \times 2x$

$$= \pi (r^2 - x^2) \times 2x$$

$$= 2\pi x (r^2 - x^2) \quad (1)$$

(iii) $V = 2\pi r^2 x - 2\pi x^3$

$$\frac{dV}{dx} = 2\pi r^2 - 2\pi \times 3x^2$$

$$= 2\pi r^2 - 6\pi x^2$$

Marker's comments:

c(i), Students forgot to give reasoning however no marks were deducted

c(ii), Very well done.

c(iii), Most students had trouble with this part.
Students who did $\frac{dV}{dr}$ were not given any marks.

Students who did $\frac{dV}{dx}$ were taken marks off if they did not do the double derivative test or Equivalent to show that maximum occurs at $x = \frac{r}{\sqrt{3}}$.

$V' = 0 \Rightarrow 2\pi r^2 = 6\pi x^2$

$x^2 = \frac{r^2}{3}$

$x = \frac{r}{\sqrt{3}} \quad (\because x > 0)$

$V'' = 0 - 6\pi \times 2x$

$= -12\pi x$

when $x = \frac{r}{\sqrt{3}}$, $V'' = -12\pi \times \frac{r}{\sqrt{3}} < 0$

Markers comments:

$\therefore$ V is maximum when

$x = \frac{r}{\sqrt{3}}$

Maximum volume

$= 2\pi r^2 x - 2\pi x^3$

$= 2\pi r^2 \times \frac{r}{\sqrt{3}} - 2\pi \times \frac{r^3}{3\sqrt{3}}$

$= \frac{2\pi r^3}{\sqrt{3}} - \frac{2\pi r^3}{3\sqrt{3}}$ (3)

$= \frac{6\pi r^3 - 2\pi r^3}{3\sqrt{3}}$

$= \frac{4\pi r^3}{3\sqrt{3}}$

Question 8 (13 marks)

(a) $\int_0^a f(x)\,dx = 0$

$\int_0^a (x^2 - 4)\,dx = 0$

$\left[\frac{x^3}{3} - 4x\right]_0^a = 0$

$\frac{a^3}{3} - 4a = 0$

$\frac{a^3}{3} = 4a$ (3)

$a^3 = 12a$

$a^2 = 12 \quad (\because a > 0)$

$a = \sqrt{12} \quad (\because a > 0)$

$= 2\sqrt{3}$

b) $\frac{dy}{dx} = 6x^2 + kx - 12$

When $x = -1$, $\frac{dy}{dx} = 0$

$6 - k - 12 = 0$

$k = -6$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$y = 6\frac{x^3}{3} - \frac{6x^2}{2} - 12x + C$

$y = 2x^3 - 3x^2 - 12x + C$

Markers comments:

- A large number of students did not realise that $\int_0^a f(x)\,dx = 0$
- got full marks for correct answer and supporting working out but more mistakes were made

- mostly well done
- Some silly mistakes with signs.

When $x=-1$, $y=1$

$1 = -2-3+12+C$

$C = -6$

$y = 2x^3 - 3x^2 - 12x - 6$ (3)

(c) $h = \frac{2-0}{4} = \frac{1}{2}$

0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
1	$\sqrt{3}$	3	$\sqrt{27}$	9

$\int_0^2 3^x \, dx$ (3)

$\doteq \frac{1}{4}\{1+9+2(\sqrt{3}+3+\sqrt{27})\}$

$= 29.856/4 = 7.464$ (to 3dp)

(d) $A = \int_0^2 -x^2 + \frac{1}{2}\times 1\times 4 + \frac{1}{2}\times 1\times 4$

$= \left[\frac{x^3}{3}\right]_0^2 + 4$

$= \frac{1}{3}(8-0)+4$ (4)

$= \frac{8}{3} + 4$

$= \frac{20}{3}$ or $6\frac{2}{3}$ square units

Marker's comments:

- mostly well done
- some students (small number) need to review the trapezoidal rule.
- A significant number of students did not use areas of Δs and wasted time but often ended up with the correct answer.

Question 9 (13 marks)

(a)(i) Area $A = \int_1^2 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_1^2$ (2)

$= \frac{1}{3}(8-1) = \frac{7}{3}$ square units

(ii) Area $B = -\int_1^2 (x^2 - 2x)\,dx$

$= \int_1^2 (2x - x^2)\,dx$

$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_1^2$

$= \left[x^2 - \frac{x^3}{3}\right]_1^2$ (3)

$= \left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right)$

$= 4 - \frac{8}{3} - 1 + \frac{1}{3}$

$= 3 - \frac{8}{3} + \frac{1}{3}$

$= 3 - \frac{7}{3} = \frac{9-7}{3}$

$= \frac{2}{3}$ square units

(iii) Area C =

$= \int_2^3 (x^2 - (x^2 - 2x))\,dx$

Markers comments:

Mostly done well.

Some students found area (A+B) and then subtracted A from it. More work for no reason.

$$= \int_2^3 (x^2 - x^2 + 2x)\, dx$$

$$= \int_2^3 2x\, dx = \left[\frac{2x^2}{2}\right]_2^3$$

(3)

$$= \left[x^2\right]_2^3$$

$$= 9 - 4 = \underline{5 \text{ square units}}$$

(b) $(y-2)^2 = x + 4$ — (1)

$$y = \frac{x}{2} \quad — (2)$$

substitute (2) in (1)

$$\left(\frac{x}{2} - 2\right)^2 = x + 4$$

$$\frac{x^2}{4} - 2x + 4 = x + 4$$

$$\frac{x^2}{4} - 3x = 0$$

$$x^2 - 12x = 0$$

$$x(x - 12) = 0$$

$$x = 12 \quad (\because x \neq 0)$$

$$y = \frac{x}{2} = \frac{12}{2} = 6$$

(2)

$\underline{B\ (12, 6)}$

Markers comments:

Mostly done well.

Lot of students made mistakes in finding the point of intersection.

(ii) Area

$$= \int_0^6 (2y - (y-2)^2 + 4)\,dy$$

$$= \int_0^6 (6y - y^2)\,dy$$

$$= \left[\frac{6y^2}{2} - \frac{y^3}{3}\right]_0^6 \quad (3)$$

$$= \left[3y^2 - \frac{y^3}{3}\right]_0^6$$

$$= \left(3 \times 36 - \frac{216}{3}\right) - 0$$

$$= 108 - 72$$

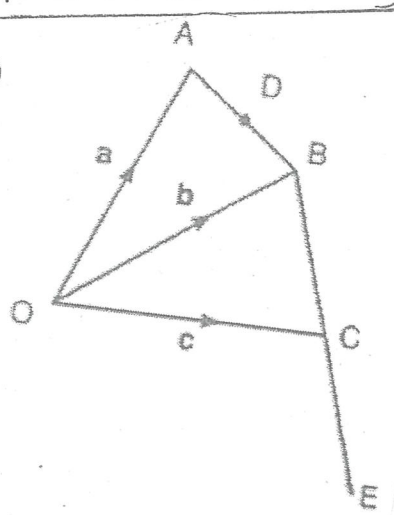
$$= \underline{36 \text{ square units}}$$

Marker's comments:

Mostly done well.

Question 10 (12 marks)

(a)



(i) $\overrightarrow{AD} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

(iii) $\overrightarrow{BC} = \underset{\sim}{c} - \underset{\sim}{b}$ (6)

(iv) $\overrightarrow{BE} = 2(\underset{\sim}{c} - \underset{\sim}{b})$

(v) $\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$

$= \underset{\sim}{b} + 2(\underset{\sim}{c} - \underset{\sim}{b})$

$= 2\underset{\sim}{c} - \underset{\sim}{b}$

(vi) $\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD}$

$= 2\underset{\sim}{c} - \underset{\sim}{b} - \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

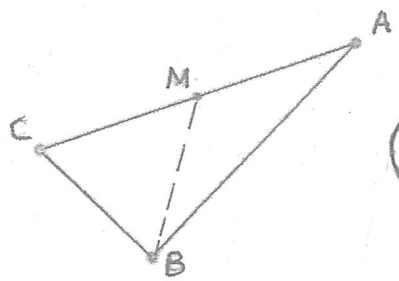
$= 2\underset{\sim}{c} - \frac{\underset{\sim}{a}}{2} - \frac{3\underset{\sim}{b}}{2}$

Marker's comments:

These questions were all very well done overall.

Please note: Students should simplify expressions. Not penalised this time.

(b) (i)



(1)

(ii) $\overrightarrow{BA} = \overrightarrow{BM} + \overrightarrow{MA}$

$= \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{BC} = \overrightarrow{BM} + \overrightarrow{MC}$ (2)

$= \underset{\sim}{b} + \underset{\sim}{a}$

(iii) $\overrightarrow{BC} \cdot \overrightarrow{BA} = (\underset{\sim}{b} + \underset{\sim}{a}) \cdot (\underset{\sim}{b} - \underset{\sim}{a})$

$= \underset{\sim}{b} \cdot \underset{\sim}{b} - \underset{\sim}{b} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{a}$

$= |\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2$ must mention

$|\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2 = 0$ $\left(\because BC \perp BA \text{ or } \overrightarrow{BC} \cdot \overrightarrow{BA} = 0\right)$

$|\underset{\sim}{b}| = |\underset{\sim}{a}|$ (3)

$|\overrightarrow{BM}| = |\overrightarrow{AM}|$

So $|\overrightarrow{BM}| = |\overrightarrow{AM}| = |\overrightarrow{MC}|$ ($\because$ M is the midpoint)

Marker's comments

well done.

well done

Reasonably well done. However, reasoning was sometimes lacking and this was penalised.
- some not using appropriate notation for magnitude.
- Note $\underset{\sim}{b} \neq \underset{\sim}{a}$ but $|\underset{\sim}{b}| = |\underset{\sim}{a}|$

Question 11 (10 marks)

(a) $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$, $\underset{\sim}{b} = \underset{\sim}{i} - 4\underset{\sim}{j}$

$$\text{proj}_{\underset{\sim}{b}} \underset{\sim}{a} = (\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}})\,\hat{\underset{\sim}{b}}$$

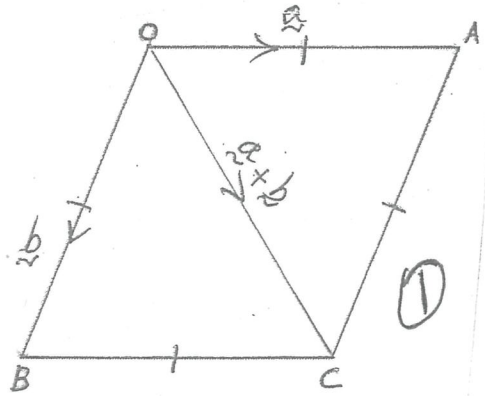
$$= \left((2\underset{\sim}{i} + 3\underset{\sim}{j}) \cdot \left(\frac{\underset{\sim}{i} - 4\underset{\sim}{j}}{\sqrt{1^2 + 4^2}} \right) \right) \frac{\underset{\sim}{i} - 4\underset{\sim}{j}}{\sqrt{1^2 + 4^2}}$$

$$= \left(\frac{(2 \times 1) + (3 \times -4)}{\sqrt{17}} \right) \frac{\underset{\sim}{i} - 4\underset{\sim}{j}}{\sqrt{17}}$$

$$= \frac{-10}{17} (\underset{\sim}{i} - 4\underset{\sim}{j}) \quad (3)$$

$$= \frac{-10}{17}\underset{\sim}{i} + \frac{40}{17}\underset{\sim}{j}$$

(b)



(1)

(ii) $\cos \angle AOC = \dfrac{\underset{\sim}{a} \cdot (\underset{\sim}{a} + \underset{\sim}{b})}{|\underset{\sim}{a}||\underset{\sim}{a} + \underset{\sim}{b}|}$

$$= \frac{\underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|\,|\underset{\sim}{a} + \underset{\sim}{b}|} = \frac{|\underset{\sim}{a}|^2 + \underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|\,|\underset{\sim}{a} + \underset{\sim}{b}|} \quad (2)$$

Marker's comments:

Some students wrote $\frac{-10}{\sqrt{17}}\underset{\sim}{i} + \frac{40}{\sqrt{17}}\underset{\sim}{j}$

representing $\underset{\sim}{a} + \underset{\sim}{b}$ as $\overrightarrow{BA}$

(iii) $\cos \angle BOC = \dfrac{\underset{\sim}{b} \cdot (\underset{\sim}{a}+\underset{\sim}{b})}{|\underset{\sim}{b}|\,|\underset{\sim}{a}+\underset{\sim}{b}|}$

$= \dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b}}{|\underset{\sim}{b}|\,|\underset{\sim}{a}+\underset{\sim}{b}|} = \dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a} + |\underset{\sim}{b}|^2}{|\underset{\sim}{b}|\,|\underset{\sim}{a}+\underset{\sim}{b}|}$

$\cos \angle AOC = \cos \angle BOC$

$(\because |\underset{\sim}{a}| = |\underset{\sim}{b}|$ and $\underset{\sim}{a}\cdot\underset{\sim}{b} = \underset{\sim}{b}\cdot\underset{\sim}{a})$

(4)

$\angle AOC = \angle BOC$

$(\because \cos x$ is one-one function for $0 \le x \le \pi)$

$\therefore$ the diagonal OC bisects $\angle AOB$.

Markers comments:

Some students are finding expressions for $\angle AOB$ and trying to prove $2\angle AOC = \angle AOB$

They were not successful in proving the result.